Exercise 1.15

A useful and easy-to-remember approximate value for the number of seconds in a year is $\pi \times 10^7$. Determine the percent error in this approximate value. (There are 365.24 days in one year.)

Solution

Start by finding the exact number of seconds in one year.

$$365.24 \; \cancel{\text{days}} \times \frac{24 \; \cancel{\text{hours}}}{1 \; \cancel{\text{day}}} \times \frac{60 \; \cancel{\text{min}}}{1 \; \cancel{\text{hour}}} \times \frac{60 \; \text{s}}{1 \; \cancel{\text{min}}} \approx 3.1557 \times 10^7 \; \text{s}$$

Use the following formula for the percent error.

$$\text{percent error} = \frac{\text{Observed Value} - \text{True Value}}{\text{True Value}} \times 100\%$$

$$= \frac{\pi \times 10^7 - 3.1557 \times 10^7}{3.1557 \times 10^7} \times 100\%$$

$$\approx -0.446\%$$

What this means is that $\pi \times 10^7$ is less than one percent below the actual value. It's a very good approximation.